Complete Algorithms for Reorienting Polyhedral Parts using a Pivoting Gripper

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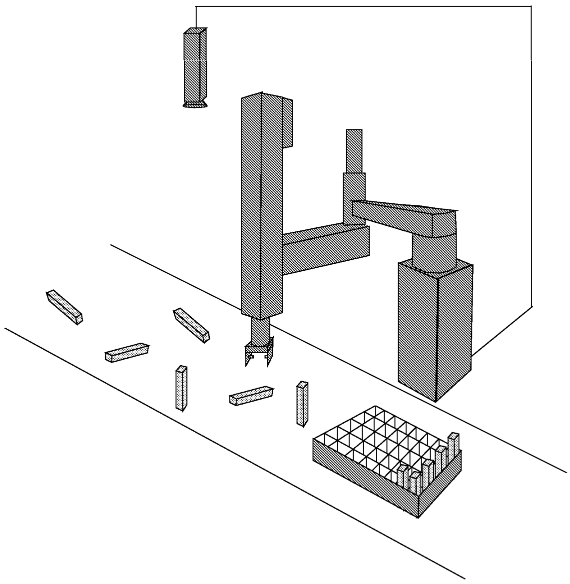

Figure 1: The pivoting gripper mounted on a SCARA arm.

1 Introduction

Achieving a desired spatial configuration of a part is a fundamental issue in robotics. In industrial applications, a familiar task is that of feeding parts: bringing parts into a desired position and orientation (pose). To rapidly feed a stream of industrial parts arriving on a conveyor belt, the vision-based system proposed by Carlisle *et. al.* [1] uses a SCARA-type arm with only 4 DoF due to cost, accuracy, and speed requirements. However, such arms can only reorient parts about the vertical axis due to kinematic limitations (see Fig. 1).

Contact between a part and a supporting plane only occurs along its convex hull. When rotations and translations in the plane are ignored, the part generally assumes one of a finite number of stable configurations [3]. In this communication, we will consider computing a sequence of pivoting actions that will move a *polyhedral* part with n faces from an initial stable configuration $\hat{s}$ to a final stable configuration $\hat{\mathbf{f}}$. The decision question is whether or not a *single* pivot action can accomplish this task: we give a $O(n \log n)$ time solution.

It may not be possible to move the part between an arbitrary pair of stable configurations in a single pivot action; in general a sequence of pick-pivot-place operations may be necessary. Therefore, we consider computing the complete graph of possible transitions and give an algorithm that runs in $O(m^2 n \log n)$ time, m being the number of stable configurations. A path through this transition graph represents a plan which moves the part from some initial to final configuration. The algorithm is complete in that whenever a path of pivot actions exists, each conforming with the gripper accessibility and friction constraints, it will be found.

See the complete paper [4] for the details which also includes a generalization that considers "capture regions" around stable configurations.

2 Problem Statement

We assume *(i)* The worktable is a flat plane orthogonal to gravity at a known height; *(ii)* The parallel-jaw gripper is able to translate with 3 DoF and to rotate about the gravity vector; *(iii)* The gripper has a passive degree of freedom – a pivot axis parallel to the support plane; and iv The gripper makes "hard contact" with the part – point contact with friction which offers no static resistance to rotation about the pivot axis.

Fig. 1 shows the robot work cell. The **input** to the algorithm is: A polyhedral part $\mathcal{P}$ stored as a boundary representation (B-rep), its center of gravity, c, which is taken to be the origin of the coordinate system used to define the B-rep, and the coefficient of static friction μ_{static}.

The **output** is a transition graph whose nodes are the stable configurations, or faces F_i of the convex hull. The arcs between nodes describe points on the part corresponding to grasp axes that will rotate the part from one stable face to another.

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3 Computing the Transition Graph

First compute the convex hull $\mathcal{H}$ for the polyhedron $\mathcal{P}$. A face of $\mathcal{H}$ is stable when the projection of the center of gravity in the normal direction onto the face lies within the face; the stable faces become the nodes of the transition graph. For every ordered pair of stable faces of $\mathcal{H}$, whose normals are given by $\hat{\mathbf{s}}$ and $\hat{\mathbf{f}}$, determine the set of grasp points (if there are any) that will pivot the part to $\hat{\mathbf{f}}$ as described below. The direction of the grasp axis is given by:

$$\hat{\mathbf{a}} = \frac{\hat{\mathbf{s}} \times \hat{\mathbf{f}}}{|\hat{\mathbf{s}} \times \hat{\mathbf{f}}|}. \tag{1}$$

Note: $\hat{\mathbf{a}}$ is undefined when $\hat{\mathbf{s}}$ and $\hat{\mathbf{f}}$ are parallel or anti-parallel. In these cases, precise pivot actions are unnecessary or impossible. The parametric equation of the family of grasp axes indexed by λ is:

$$\mathbf{a}_\lambda(t) = t\hat{\mathbf{a}} - \lambda\hat{\mathbf{f}}, \tag{2}$$

where $\lambda > 0$ can be interpreted as the distance from the center of gravity to the axis. Thus, the grasp axis must lie in the half-plane, $\mathcal{A}$, spanned by $\hat{\mathbf{a}}$ and $-\hat{\mathbf{f}}$.

1. Determine the half-plane $\mathcal{A}$ of grasp axes which will successfully pivot $\mathcal{P}$ according to Eq. (2) and the direction of the grasp axis $\hat{\mathbf{a}}$ from Eq. (1).
2. Compute the intersection of $\mathcal{A}$ with $\mathcal{P}$ which yields a collection of intersection polygons $\mathbf{P}$ in the grasp plane.
3. In the direction $\hat{\mathbf{f}}$ within the grasp plane, compute the upper $\mathcal{U}$ and lower $\mathcal{L}$ envelope of the polygon(s) $\mathbf{P}$. The upper (lower) envelope is the portion of $\mathbf{P}$ visible from infinitely far away along $+\hat{\mathbf{a}}$ $(-\hat{\mathbf{a}})$. Each envelope is a function of λ, and the edges of the envelope are ordered by increasing λ. The importance of points on these envelopes is that they are *accessible* to a gripper linearly approaching the part along the grasp axis.
4. For each edge of $\mathcal{U} \cup \mathcal{L}$ whose corresponding face has surface normal $\hat{\mathbf{n}}$, determine if the face can be grasped by a point contact with friction in the direction $\hat{\mathbf{a}}$ according to: $||\hat{\mathbf{a}} \cdot \hat{\mathbf{n}}|| \leq \cos\alpha$. Here α is the *friction angle* computed from $\tan(\alpha) = \mu_{\mathrm{static}}$.
5. Merge the two sorted envelopes $\mathcal{U}$ and $\mathcal{L}$ into a set $\Lambda = \cup\Lambda_i$ where each Λ_i is a closed interval of λ. Associated with each interval is the pair of functions $u(\lambda)$ and $l(\lambda)$ which return the grasp points.
6. If $\Lambda \neq \emptyset$, create an arc in the transition graph between $\hat{\mathbf{s}}$ and $\hat{\mathbf{f}}$.

The complexity of the algorithm is dominated by the construction of the envelopes which takes $O(n \log n)$ time per iteration [2]; the rest of the steps have linear complexity. Since there are $O(m^2)$ pairs of stable configurations, the complexity of constructing the entire transition graph is $O(m^2 n \log n)$. For star-shaped (wrt the center of gravity) polyhedra, this reduces to $O(m^2 n)$ because the intersections computed in Step 2 will each consist of a single star-shaped polygon.

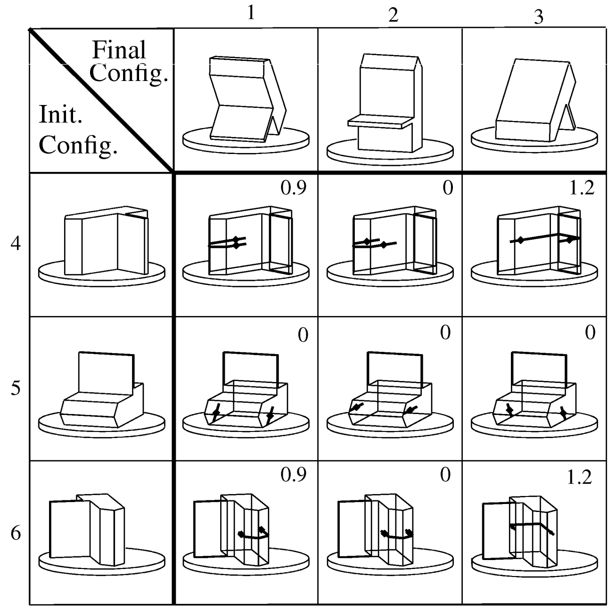

Figure 2: The partial matrix of transitions for the part with six stable configurations. Cell (i, j) indicates the family of accessible pivot grasps that will move configuration i to configuration j; the optimal grasp (requiring minimal μ_{static}) from among this family is shown as a pair of disks. Numbers in the upper right-hand-corner of each cell indicate the minimal required coefficient of friction.

Implementation: The algorithm for planning pivot actions was implemented in the Symbolic Computing System *Maple V*. The choice of Maple was made because several primitive geometric tests and computations are built in with Maple's *geom3d* package. As an example, consider Fig. 2. The part has $n = 11, m = 6$. The entire transition graph, a fourth of which is shown in the figure, was computed in 28 seconds on a Silicon Graphics workstation (R4400 processor running at 150 MHz, 96.5 SPECfp92, 90.4 SPECint92).

References

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