A. Properties of $L_t(C, z, S(x))$ loss function

Lemma 1. The loss function

$$L_t(C, z, S(x)) = \sum_{j,k=1}^{M} C_{z,j} e^{S_j(x) - S_k(x)} \quad (1)$$

is Classification Calibrated.

Proof. Using (1), risk of classification is

$$R_{L_t}[S] = E_{X,Z}[L_t[C, z, S(x)]]$$

$$= \sum_{z=1}^{M} \eta_z(x) L_t(C, z, S(x)) \quad (2)$$

$$= \sum_{z=1}^{M} \eta_z(x) \sum_{j,k=1}^{M} C_{z,j} e^{S_j(x) - S_k(x)} \quad (3)$$

$$= \sum_{z=1}^{M} \sum_{j,k=1}^{M} \eta_z(x) C_{z,j} e^{S_j(x) - S_k(x)} \quad (4)$$

To find the optimal scores, $S^*(x)$, we start by setting

derivatives to zero, where

$$0 = \frac{\partial R_{L_t}[S]}{\partial S_i(x)} = \sum_{z=1}^{M} \eta_z(x) C_{z,i} e^{S_i(x)} \sum_{k=1}^{M} e^{-S_k(x)}$$

$$- e^{-S_i(x)} \sum_{z=1}^{M} \sum_{j=1}^{M} \eta_z(x) C_{z,j} e^{S_j(x)} \quad (5)$$

results in

$$e^{-2S_i(x)} = \sum_{z=1}^{M} \eta_z(x) C_{z,i} \frac{\sum_{k=1}^{M} e^{-S_k(x)} \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} e^{S_j(x)}}{\sum_{z,j=1}^{M} \eta_z(x) C_{z,j} e^{S_j(x)}} \quad (6)$$

$$e^{-2S_i(x)} \sum_{z=1}^{M} \eta_z(x) C_{z,i} = \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} e^{S_j(x)} \quad (7)$$

Assuming $\sum_{i=1}^{M} S_i(x) = 0$, and defining

$$\psi(i) = \sum_{z=1}^{M} \eta_z(x) C_{z,i} \quad (8)$$

results in

$$S^*_i(x) = -\frac{1}{2} \log(\psi(i)) + \frac{1}{2M} \sum_{j=1}^{M} \log(\psi(j)) \quad (9)$$

Therefore $S^*_i(x)$ will be inversely proportional to Bayes
cost of $i^{th}$ class and thus (1) will be classification

Lemma 2. $L_t(C, z, S(x))$ is not guess-averse.

Proof. The proof is based on a counter example. Assume a
cost insensitive problem i.e $C_{i,j} = 1 \; i \neq j$, with $M = 3$
and $S(x) = [3, 0, -3]$ where example $x$ belongs to the first
class. In this case $S(x)$ results in the correct prediction but
its loss is greater than random guessing since $L(C, 1, 0) = 6 < L(C, 1, S(x)) \approx 22.19$. Therefore (1) is not guess-

B. Comparing Guess-aversion and
c-calibration

We start with following lemma that shows that c-
calibration (Vernet et al., 2011) implies guess-aversion.

Lemma 3. If a loss function $L(C, z, S(x))$ is c-calibrated, then it will be guess-averse.

Proof. If $L(C, z, S(x))$ is c-calibrated, then according to
c-calibration definition

$$\forall s_1 \in S_z \quad \forall s_2 \notin S_z, L(C, z, s_1) < L(C, z, s_2). \quad (10)$$

In addition note that, $0 \notin S_z$ and using (11)

$$\forall s_1 \in S_z \quad L(C, z, s_1) < L(C, z, 0). \quad (11)$$

which is definition of guess-aversion.
Lemma 4. If a loss function $L(C, z, S(x))$ is guess-averse, then it may not be c-calibrated.

Proof. The proof is based on counter example. Consider the cost-insensitive GLL-loss of Figure 3-b. Since it satisfies Lemma 1 in the paper this loss is guess-averse. However, for sufficiently small $\epsilon > 0$, the set $\mathcal{A}_c = \{S|S \in S_2, L(C, 1, S) < L(C, 1, 0) - \epsilon\}$ is non-empty. Similarly, since the loss surface is continuous and smooth in Figure 3-b, there exists a point $p_0 \in \mathcal{S}_1$ such that $L(C, 1, p_0) > L(C, 1, 0) - \epsilon$. Therefore for any $q_0 \in \mathcal{A}_c$, $L(C, 1, p_0) > L(C, 1, q_0)$ which is contradictory to c-calibration, since $p_0$ results in correct classification and $q_0$ does not.

C. Properties of the Generalized Exponential Loss

Lemma 5. If $C_{i,j} \geq 0 \forall i, j = 1 \ldots M$ and $\exists i, j : C_{i,j} > 0$ then

\[ R_{L^{id,exp}} = E_X \{ L^{id,exp}(C, z, S(x)) \} \]

\[ = \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} e^{S_j(x)-S_z(x)}, \quad (13) \]

is strictly convex with respect to $S(x) \in \mathbb{R}^M$.

Proof. Denoting $\beta_{i,j} = 1_i - 1_j$, we start by computing first and second order derivatives,

\[ \frac{\partial R_{L^{id,exp}}}{\partial S} = \frac{\partial}{\partial S} \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} e^{S_j(x)-S_z(x)} \]

\[ = \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} \beta_{i,j} e^{S_j(x)} \]

\[ \frac{\partial^2 R_{L^{id,exp}}}{\partial S^2} = \frac{\partial}{\partial S} \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} \beta_{i,j} e^{S_j(x)} \]

\[ = \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} [\beta_{j,z} \beta_{i,j}^T] e^{S_j(x)}, \quad (14) \]

where we omitted $x$ for simplicity. Note that $[\beta_{j,z} \beta_{i,j}^T]$ is positive definite for all $z, j$, moreover $C_{i,j} \geq 0 \forall i, j = 1 \ldots M$ and $\exists i, j : C_{i,j} > 0$. Therefore the hessian is a sum of positive definite matrices, and is a positive definite matrix. Therefore $R_{L^{id,exp}}$ is strictly convex.

Lemma 6. If the cost matrix, $C$, is symmetric then the minimizer of $R_{L^{id,exp}(C, z, S(x))}$, (13), is independent of $C$.

Proof. We start by setting (14) to zero, therefore

\[ \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} \beta_{i,j} e^{S_j(x)} = \sum_{z,j=1}^{M} \eta_z(x) C_{z,j} \beta_{i,j} e^{S_j(x)} \]

and thus

\[ \sum_{j=1}^{M} \eta_k C_{k,j} e^{S_j(x)-S_k(x)} = \sum_{z=1}^{M} \eta_z C_{z,k} e^{S_k(x)-S_z(x)}. \quad (17) \]

However note that when $C$ is symmetric,

\[ S_k(x) = \frac{1}{2} \log(\eta_k(x)) - \frac{1}{2M} \sum_{j=1}^{M} \log(\eta_j(x)) \]

satisfies (17). This is because $e^{S_j(x)-S_k(x)} = \frac{\sqrt{\eta_j}}{\sqrt{\eta_k}}$ and thus left and right sides of (17) become equal. Therefore (18) is a minimizer of $R_{L^{id,exp}(C, z, S(x))}$. In addition according to lemma (5), $R_{L^{id,exp}}$ is strictly convex and thus (18) will be the unique minimizer.

D. MLC - Cost Matrix

The cost matrix for the Moorea Labelled Corals dataset is shown in Table 1. The costs are set with an eco logic application in mind. There, the most important goal is a binary estimate of the amount of corals versus everything else. Thus, the cost of confusion between the coral genera (classes 5-9) and the non-corals (classes 1-4) is set to a high value, 4. Cost of confusion among corals is low, 1, and similarly for cost of confusion among algae (classes 1-3). Finally, confusion between any algae and the sand class is worse than confusion within algae, but not as bad as confusion to (or from) corals. These values are set to 2.

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structured SVM (Tsochantaridis et al., 2004) solves

\[
\begin{align*}
\min_{w, \epsilon} & \frac{1}{2} \|w\|^2 + \sum_{i=1}^{n} \epsilon_i \\
\text{s.t.,} & \quad \forall i, z \in Z, \quad S_z(x_i) + C_{z, z} \leq S_z(x_i) + \epsilon_i \\
& \quad \epsilon_i \geq 0 \forall i,
\end{align*}
\]  

(21)

where

\[ S_z(x) = \langle w, \Psi(x, z) \rangle \]  

(22)

is the score of structure \( Y_z \) for the example \( x \). \( \Psi(x, z) \) is a feature vector extracted with respect to structure \( Y_z \), and \( C_{z, z} \geq 0 \) is the cost of assigning structure \( Y_z \) instead of the true structure \( Y_z \).

An equivalent way of writing (21) is

\[
\min_{w} \frac{\lambda}{2} \|w\|^2 + \sum_{i} L_H(C, z_i, S(x_i))
\]  

(23)

where

\[ L_H(C, z_i, S(x_i)) = \max_k [C_{z, k} + (S_k(x) - S_z(x))] \]

\[ < \max_k (C_{z, k}) \]

\[ = L_H(C, z, 0). \]

Therefore if \( S(x) \in S_z \), then \( L_H(C, z, S(x)) < L_H(C, z, 0) \) and thus \( L_H \) is guess-averse.

\textbf{References}

